Measuring with strain gauges

Formulary for the elementary load cases

Stand: 17.03.2024, Kab.

1. bending (rectangular cross-section)

Calculation of the bending moment from the measured strain or from the measured bridge detuning

The maximum stress $_{\sigma b}$ on the edge fibre results from the bending moment $_{Mb}$ and the section modulus $_{Wb}$ against bending:

$$\sigma_b = \frac{M_b}{W_b} \quad \text{(eq. 1.1)}$$

The following applies to rectangular cross-sections with beam width b and beam height h:

$$W_b = \frac{b h^2}{6} \quad \text{(eq. 1.2)}$$

With Hooke's law :

$$\sigma = E \cdot \epsilon \quad \text{(eq. 1.3)}$$

and equations 1) and 2) are calculated for the moment from the measured strain on the surface of a bending beam with a rectangular cross-section:

$$M_b = \epsilon \cdot E \cdot \frac{b h^2}{6} \quad \text{(eq. 1.4)}$$

With the linearised bridge equation

$$\frac{\Delta Ud}{Us} = \frac{1}{4} \cdot \left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4} \right) \quad \text{(eq. 1.5)}$$

and the relationship between strain and resistance change for the strain gauge

$$\frac{\Delta R}{R} = k \cdot \epsilon \quad \text{(eq. 1.6)}$$

applies to a full bridge with$\varepsilon 1 = \varepsilon 3 = -\varepsilon 2 = -\varepsilon 4 = \varepsilon$

$$\frac{\Delta U_d}{U_s} = \frac{1}{4} (4 k \epsilon) = k \cdot \epsilon \quad \text{(eq. 1.7)}$$

with eq. 1.7 in eq. 4 you get:

$$M_b = \frac{\Delta U_d}{U_s} \cdot \frac{1}{k} \cdot E \cdot \frac{b h^2}{6}$$ eq. 1.8

2. bending (cylindrical cross-section)

For cylindrical cross-sections with diameter d, the following applies to the moment of resistance against bending:

$$W_b = \frac{\pi d^3}{32}$$ eq. 2.1

Analogue to Eq. 1.4, the relationship between moment and strain is obtained:

$$M_b = \epsilon \cdot E \cdot \frac{\pi d^3}{32}$$ eq. 2.2

For the full bridge with 4 active grids, the result is analogous to Eq. 1.8:

$$M_b = \frac{\Delta U_d}{U_s} \cdot \frac{1}{k} \cdot E \cdot \frac{\pi d^3}{32}$$ eq. 2.3

For the quarter bridge, due to

$$\frac{\Delta U_d}{U_s} = \frac{1}{4} k \cdot \epsilon$$ eq. 2.4

$$M_b = \frac{\Delta U_d}{U_s} \cdot \frac{4}{k} \cdot E \cdot \frac{\pi d^3}{32}$$ eq. 2.5

3. torsion

Calculation of the torsional moment from the measured strain or from the measured bridge detuning

The maximum shear stress $_{\tau t}$ on the edge fibre results from the torsional moment $_{Mt}$ and the moment of resistance $_{Wt}$ against torsion

$$\tau_t = \frac{M_t}{W_t}$$ (eq. 9)

For the cylindrical cross-section, the section modulus Wt is equal to the polar section modulus Wp.
The following applies to the solid cylinder:

$$W_t = \frac{\pi d^3}{16} = W_p \quad \text{(Gl 10a)}$$

The following applies to the hollow cylinder:

$$W_t = \frac{\pi(d_a^4 - d_i^4)}{16 d_a} = W_p \quad \text{(eq. 10b)}$$

With Hooke's law

$$\tau = G \cdot \gamma \quad \text{(eq. 11)}$$

and eq. 9 and eq. 10B the moment M_T is calculated from the shear Y

$$M_t = \gamma \cdot G \cdot W_p = \gamma \cdot G \cdot \frac{\pi(d_a^4 - d_i^4)}{16 d_a} \quad \text{(eq. 12)}$$

Only the strain ε can be measured with the strain gauge, not the shear Y.
The relationship between shear and strain applies under a measuring direction of α=45° to the longitudinal axis:

$$\epsilon_{45} = \gamma / 2 \quad \text{(eq. 13) (veq. Anhang, 2)}$$

The shear modulus G can be derived from the modulus of elasticity E and the transverse contraction coefficient v:

$$G = \frac{E}{2 \cdot (1+\nu)} \quad \text{(eq. 14)}$$

with eq. 13 and eq. 14 in eq. 12results in the required relationship between strain and torsional moment

$$M_t = 2 \cdot \epsilon_{45} \cdot \frac{E}{2 \cdot (1+\nu)} \frac{\pi(d_a^4 - d_i^4)}{16 d_a} = \epsilon_{45} \cdot \frac{E}{(1+\nu)} \frac{\pi(d_a^4 - d_i^4)}{16 d_a} = \frac{\Delta U_d}{U_s} \cdot \frac{1}{k} \cdot \frac{E}{(1+\nu)} \frac{\pi(d_a^4 - d_i^4)}{16 d_a} \quad \text{(eq. 15)}$$

By comparing eq. 15 with eq. 11 and eq. 12 and from the (Mohr's stress circle for the stress case of torsion)can also be found:

$$\tau = \frac{E}{1+\nu} \cdot \epsilon_{45} = \sigma_1 = -\sigma_2 \quad \text{(Gl 16)}$$

4. Axialkraft

Calculation of the axial force from the measured elongation or from the measured bridge detuning

The following applies to the mechanical stress σ_z in an axially loaded bar:

$$\sigma_z = \frac{F_z}{A} \quad \text{(eq. 4.1)}$$

The cross-sectional area A results for the cylindrical full cross-section:

$$A = \pi \frac{d^2}{4} \quad \text{(eq. 4.2)}$$

with Hooke's law :

$$\sigma = E \cdot \epsilon \quad \text{(eq. 4.3)}$$

and equations 4.1) and 4.2) are calculated for the axial force $_{Fz}$ from the measured strain on the surface of a bar with a cylindrical cross-section:

$$F_z = \epsilon \cdot E \cdot \frac{\pi d^2}{4} \quad \text{(eq. 4.4)}$$

With the linearised bridge equation

$$\frac{\Delta Ud}{Us} = \frac{1}{4} \cdot \left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4} \right) \quad \text{(eq. 4.5)}$$

and the relationship between strain and resistance change for the strain gauge

$$\frac{\Delta R}{R} = k \cdot \epsilon \quad \text{(eq. 4.6)}$$

applies to a full bridge with 2 longitudinal gratings ε1 = ε3 = ε and two Quergittern = ε2 = ε4 = -ν ε

$$\frac{\Delta U_d}{U_s} = \frac{1}{4} ((2 + 2\nu) k \epsilon) = \frac{1}{2} (1 + \nu) k \cdot \epsilon \quad \text{(eq. 4.7)}$$

with eq. 4.7 in eq. 4.4 is obtained for the full bridge:

$$F_z = \frac{\Delta U_d}{U_s} \cdot \frac{2}{1 + \nu} \cdot \frac{1}{k} \cdot E \cdot \frac{\pi d^2}{4} \quad \text{eq. 4.8}$$

and for the quarter bridge:

$$F_z = \frac{\Delta U_d}{U_s} \cdot \frac{4}{k} \cdot E \cdot \frac{\pi d^2}{4} \quad \text{eq. 4.9}$$

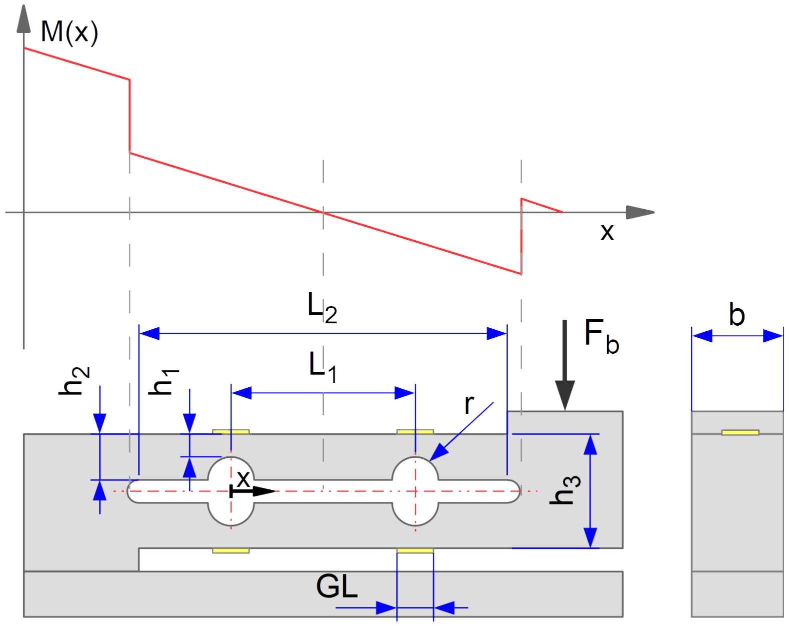
5. bending (double bending beam)

In the case of a double bending beam, a shift in the point of force application has a first approximation influence on the resulting bending moment. In the centre of the double bending beam, the moment characteristic curve has a zero crossing.

In contrast to the single bending beam, positive and negative bending moments (and therefore positive and negative strains) are present on both the top and bottom of the double bending beam.

Wheatstone's full bridge with 4 active measuring grids can be realised, for example, by equipping the double beam on one surface with two double linear strain gauges, or with four single strain gauges, two on the top and two on the bottom.

Accordingly, the bending line has an inflection point in the centre of the double beam. The inclination of the free end of the double bending beam is zero. The load causes a parallel displacement.



The equation of the bending line for the double bending beam is as follows:

$$\frac{1}{R(x)}=\frac{w''(x)}{(1+w'(x)^2)^{\frac{3}{2}}}=\frac{M(x)}{2EI_y(x)}=\frac{F_b(\frac{L_2}{2}-x)}{2EI_y(x)} \quad \text{mit} \quad I_y(x)=\frac{bh^3(x)}{12} \quad \text{(eq. 5.1)}$$

The expansion at the surfaces of each (individual) beam is:

$$\epsilon(x)=\frac{h(x)}{R(x)}\cdot\frac{1}{2} \quad \text{(eq. 5.2)}$$

Quelle [2]: Szabó, István: Einführung in die Technische Mechanik. Springer Verlag, Berlin, 1984.

If the radius of curvature R(x) in equation 5.1 is replaced by the definition of the strain in equation 5.2, the result is

$$\epsilon(x)=\frac{3M(x)}{bEh^2(x)} \quad \text{(eq. 5.3)}$$

The equation for determining the beam height at the positions of the strain gauge x=0 and $_{x=L1}$ for a given strain and given force for a double beam with two single beams without stiffeners (r=0) is with M(x=0) = F_b * L_1 / 2:

$$h_1=\sqrt{\frac{3F_b}{bE\epsilon}\cdot\frac{L_1}{2}}$$ (eq. 5.4)

The deflection of the double beam is obtained by integrating the equation for the bending line(eq. 5.1) with

$$(1+w'(x)^2)^{\frac{3}{2}}\approx 1$$ (eq. 5.6)

Let the origin of x be at the beginning of the double bar.

$$2EI_y(x)w''(x)=F_b(\frac{L_2}{2}-x)$$ (eq. 5.7)

$$2EI_y(x)w'(x)=F_b(\frac{L_2}{2}\cdot x-\frac{x^2}{2})+C_1$$ (eq. 5.8)

With the boundary condition that the inclination w'($_{x=L2}$) equals 0 at the right end of the double bar, the integration constant

C_1 = 0

$$-F_b(\frac{L_2^2}{2}-\frac{L_2^2}{2})=C_1$$; C_1 = 0; (eq. 5.9)

After the second interation applies:

$$2EI_y(x)w(x)=F_b(\frac{L_2}{2}\cdot\frac{x^2}{2}-\frac{x^3}{6})+C_2$$ (eq. 5.10)

With the boundary condition that the deflection w(x=0) equals 0 at the beginning of the double beam, the integration constant$_{C2}$ = 0 is obtained as the equation of the bending line:

$$w(x)=\frac{F_b}{2EI_y(x)}(\frac{L_2}{2}\cdot\frac{x^2}{2}-\frac{x^3}{6})$$ (eq. 5.11)

The deflection at the free endL_2 is:

$$w(L_2)=\frac{F_bL_2^3}{24EI_y(L_2)}=\frac{F_bL_2^3}{2bEh^3}$$ (eq. 5.12)

Further calculation equations for parallel guides can be found in Appendix 4).

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Appendix

1) Axial and polar moment of inertia for the circular cross-section

With the polar moment of inerita, the neutral fibre passes through a pole (a point in the centre of the circular cross-section).

With the axial area moment of inerita,the neutral fibre is an axis (an axis in the centre of the circular cross-section).

The following applies to a circular cross-section:

The polar area moment of inertia aggainst torsion is twice as large as the axial area moment of inertia against bending

$$I_p=\int_A r^2\,dA=\int_A (x^2+y^2)\,dA=\int_A x^2\,dA+\int_A y^2\,dA=I_x+I_y$$

$$W_p=\frac{I_p}{(d/2)}=\frac{1}{2}W_b=\frac{1}{2}\frac{I_b}{(h/2)}$$

2) Relationship between shear and strain

Source [4]: Agne, Klaus: Technical mechanics in precision engineering. Tasks Examples Solutions. Vieweg + Teubner verlag, Braunschweig, 1988.

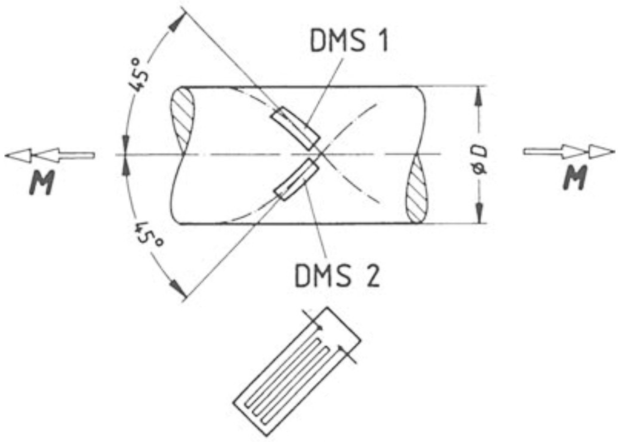


Figure 2: Torsion measurement with strain gauges

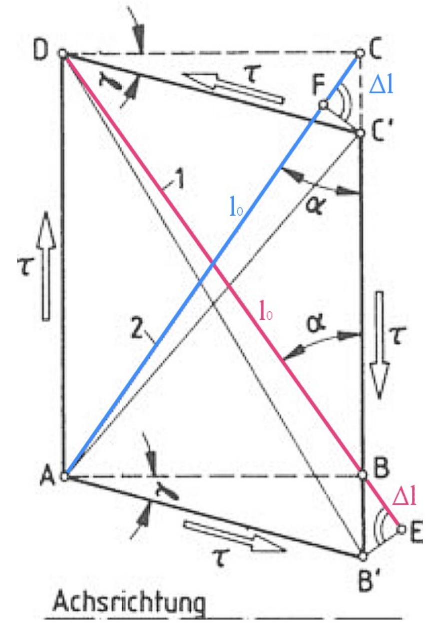


Figure 1: Relationship between shear γ and strain Δl/l in the measurement direction α

The circular arcs B'E and C'F can be assumed to be straight lines for small shears γ. The following applies for strains $_{\varepsilon 1}$ and $_{\varepsilon 2}$:

$$\epsilon_1 = \frac{\overline{BE}}{\overline{BD}};\ \epsilon_2 = \frac{-\overline{CF}}{\overline{AC}};$$

With $\gamma \approx \tan\gamma = \overline{BB'}/\overline{AB} = \overline{CC'}/\overline{CD}$ ε_1 and ε_2 :

$$\epsilon_1 = \frac{\overline{BB'}\cos\alpha}{\overline{AB}/\sin\alpha} = \frac{\overline{BB'}}{\overline{AB}}\sin\alpha\cos\alpha = \gamma\frac{1}{2}\sin 2\alpha$$

$$\epsilon_2 = \frac{\overline{CC'}\cos\alpha}{\overline{CD}/\sin\alpha} = \frac{\overline{CC'}}{\overline{CD}}\sin\alpha\cos\alpha = -\gamma\frac{1}{2}\sin 2\alpha$$

For $\alpha = 45°$

$$\epsilon_1 = \gamma/2;\ \epsilon_2 = -\gamma/2;$$

3) Resistance moments against torsion for selected cross-sections

Source [1]: Läpple, Volker: Solution manual for the introduction to strength of materials.Vieweg und Teubner Verlag, Wiesbaden, 2007.

By inserting the section moduli Wt into Eq. 15, the torsional moment Mt can be calculated from the strain or bridge detuning for non-cylindrical cross-sections.

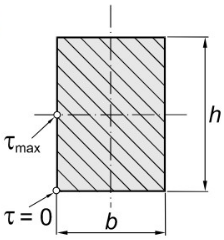
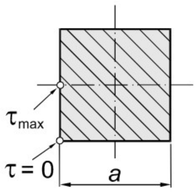
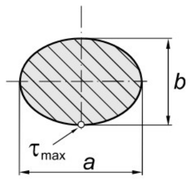
Profil	Torsionsflächenmoment I_t	Torsionswiderstandsmoment W_t
Rechteck 	$I_t = c_1 \cdot h \cdot b^3$ mit $c_1 = \frac{1}{3} \cdot \left(1 - \frac{0{,}630}{h/b} + \frac{0{,}052}{(h/b)^5}\right)$	$W_t = \frac{c_1}{c_2} \cdot h \cdot b^2$ und $c_2 = 1 - \frac{0{,}650}{1 + (h/b)^3}$
Quadrat 	$I_t = 0{,}141 \cdot a^4$	$W_t = 0{,}208 \cdot a^3$
Ellipse 	$I_t = \frac{\pi}{16} \cdot \frac{a^3 \cdot b^3}{a^2 + b^2}$	$W_t = \frac{\pi}{16} \cdot a \cdot b^2$

Figure 3: Moment of resistance Wt for rectangular cross-sections and ellipse, from [1]

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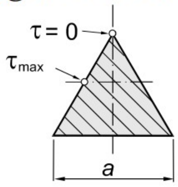
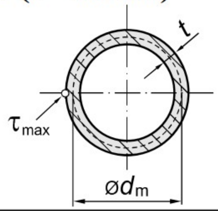
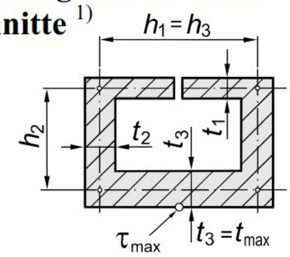
Figure 4: Moment of resistance Wt for triangular cross-sections and thin-walled, open hollow cross-sections, from [1]

Profil	Torsionsflächenmoment I_t	Torsionswiderstandsmoment W_t
Gleichseitiges Dreieck 	$I_t = \frac{a^4}{46{,}2}$	$W_t = \frac{a^3}{20}$
Dünnwandiges, geschlossenes Kreisrohr (t = konst.) 	$I_t = \frac{\pi}{4} \cdot d_m^3 \cdot t$	$W_t = \frac{\pi}{2} \cdot d_m^2 \cdot t$
Dünnwandige offene Hohlquerschnitte 1) 	$I_t = \frac{1}{3} \cdot \sum_i h_i \cdot t_i^3$	$W_t = \frac{1}{3 \cdot t_{max}} \cdot \sum_i h_i \cdot t_i^3$

4) Calculation equations for parallel guides

Source [3]: Krause, Werner: Design elements of precision mechanics. Carl Hanser Verlag, Munich, 2018.

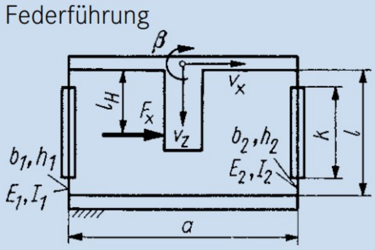
	Federführung 	
	unversteift	versteift
Auslenkung in *x*-Richtung	$v_x = \frac{F_x l^3}{24 H_0}$ $v_{x\max} \leq \frac{l^2}{3Eh}\sigma_{b\,zul}$	$v_x = \frac{F_x l^3}{24 H_0}(1-m^3)$ $v_{x\max} \leq \frac{l^2(1-m^3)}{3Eh}\sigma_{b\,zul}$
in *z*-Richtung	$v_z = \frac{F_x^2 l^5}{960 H_0^2}$	$v_z = \frac{F_x^2 l^5}{960 H_0^2}\left(1-\frac{5}{2}m^3+\frac{3}{2}m^5\right)$
Biegesteifigkeit	$H_0 = \frac{1}{2}(E_1 I_1 + E_2 I_2)$	
Axiales Flächenträgheitsmoment	$I = \frac{bh^3}{12}$	
Versteifungsverhältnis		$m = \frac{k}{l}$

Figure 5: Table 8.3.3 'Calculation equations for simple parallel guides', from [3]